8.4 Part 2

Summary:

Last time: Gauss Divergence Theorem ⑥

$$\iiint_W \operatorname{div}(F)\,dV = \iint_{\partial W} F \cdot d\vec{S}$$

inside   boundary

where:
- W is a solid in $\mathbb{R}^3$
- ∂W is oriented outwards
- F is a vector field

This Gauss divergence theorem

- is a calculational tool (choose which side is easier)
- it explains the meaning of the divergence operation.

For a vector field $F = \langle P, Q, R\rangle$ we have defined divergence by a formula $\boxed{\operatorname{div} F} = \nabla \cdot F = P_x + Q_y + R_z$.

However, if our vector field F has a physical meaning, we hope that $\operatorname{div}(F)$ does too!

Eg. F is the velocity vector field of for a flow of some quantity Q.

∴ If we do not understand the meaning of divergence from the formula - we can try to understand it using the Gauss Divergence Theorem.

$\boxed{\text{Conclusion}}$ (last time) The value of the function div(F) at a point P is

the $\boxed{\text{rate}}$ of {change / creation (accumulation)} of the quantity Q at ~~the~~ the point P.

Final project problem Explain this conclusion. (Guidance: series of questions)

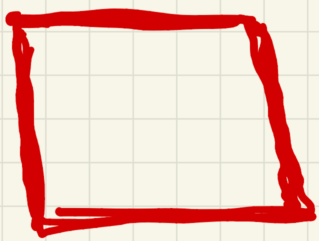
(Sources: notes
- book
- review: last week)

Heat flow

P heater
created

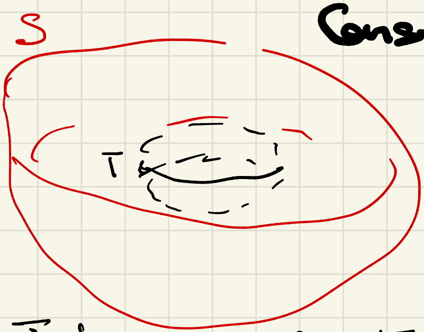
Air flow

P pressure forces air to become denser at P
accumulation



The last topic is:

D. Moving the surface of integration

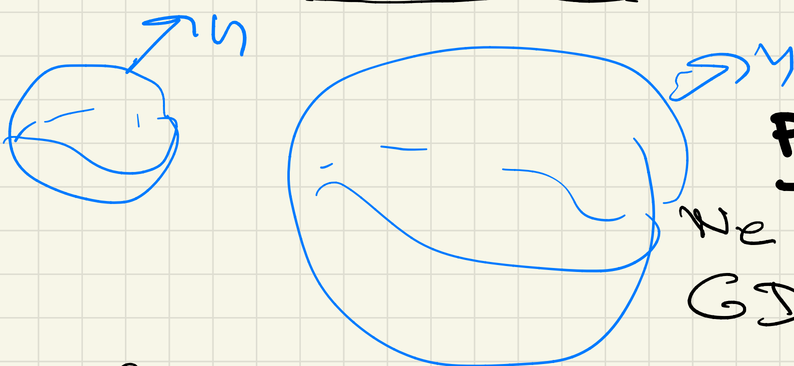


Consider two closed surfaces S and T with T lying inside S. Choose the solid W to be the space in between S and T.

T1. Let F is a vector field defined on W & such that $\operatorname{div}(F) = 0$.

Then $$\iint_S F \cdot d\vec{S} = \iint_T F \cdot d\vec{S},$$

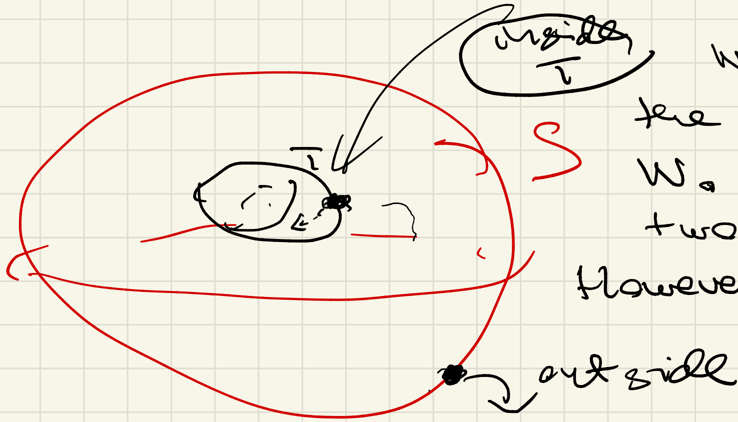
if both S & T are oriented in the same way, say outwards.



Proof. We will use GDT for W.

So,

$$\underbrace{\iiint_W \underbrace{\operatorname{div}(F)}_{=0}\, dV}_{=0} = \iint_{\partial W} F \cdot d\vec{S} \quad (?)$$



We need to understand the boundary ∂W of W. It consists of two pieces S and T. However, when we take into accent orientations we get

∂W = S with plus and T with minus:

① $\boxed{\text{boundary of } W}$ has orientation from W and it points out of W

{this is the orientation in GD [ ]

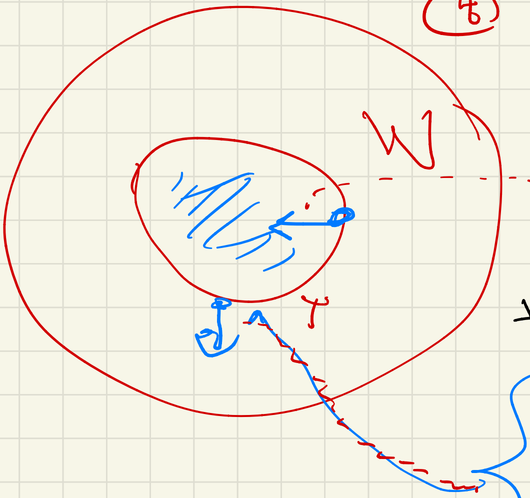
② We will

$\boxed{\text{Compare}}$ with orientations of S & T which were chosen as "out of S" and "out of T".

③ **On S**: orientation out of W
||
orientation out of S.

Because at a point p on S,

out of W = out of space enclosed by S.



(4) **Over T**

orientation out of W is the opposite of the orientation out of T, i.e. out of the solid enclosed by T.

(5) So the **Precise** formula for ∂W is

$$\partial W = \underbrace{+S}_{\text{agree}} \quad \underbrace{-T}_{\text{opposite}}$$

, when we take into account orientations.

(6) Now

$$0 = \iiint_W \operatorname{div}(F)\, dV \overset{\text{GDT}}{=} \iint_{\partial W} F \cdot d\vec{S}$$

$$= \left\{ \iint_S F \cdot d\vec{S} - \iint_T F \cdot d\vec{S} \right.$$

So:

$$\iint_T = \iint_S$$

as promised!

$\blacksquare$

Conclusion:

For surface integrals $\iint_S \vec{F} \cdot d\vec{S}$

one can move the surface S

provided that $div(\vec{F}) = 0$

between surfaces!!

R. This will be used in

Final Project!

Problem: Explain

Gauss Law

p 57